CALCULATING THE PARAMETERS OF ELECTRIC-ARC HEATERS WITH GAS-STABILIZED ARCS

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We used a turbulent model of a longitudinally streamlined electric arc to derive an analytical solution for the problem of the distribution of the primary flow and discharge parameters in a cylindrical discharge channel of an electric-arc heater.

The arc heaters in which the discharge is stabilized by a gas flow and which is situated along the axis of the cylindrical channel are used extensively in various branches of scientific and engineering research. The discharge and the plasma flow are formed in the stabilization channel which is usually represented by one of the electrodes. In certain cases, the role of the stabilization channel is taken over by an interelectrode insert insulated from the electrodes. The arc begins at the inside (rod or flask-shaped) electrode and closes on the side wall of the cylindrical output electrode. Such a circuit makes some provisions for gas stabilization and arc stabilization by the walls of the discharge channel. The formulation and solution of the problem involved in determining the flow and discharge parameters involve a number of fundamental difficulties: the question of the relationship between the primary mechanisms of heat transfer in the flow has not been resolved (molecular heat conduction, turbulent mixing, radiation); no rigorous description exists of the flow and discharge patterns within the channel zone adjacent to the arc supports; the arc-shunting mechanism, basic to the determination of arc length, has not been worked out quantitatively. The determination and resolution of these problems call for extensive and rather precise experimental studies of the electrical and gasdynamic plasma-flow parameters in actual arc heaters with gas-stabilized arcs. Work along these lines is under way; however, the data derived to this time are few in number [1, 2].

To achieve theoretical calculation methods we employ a certain idealization of the flow and discharge patterns in a plasmatron. The most common example of such idealization is the channel model of an electric arc [3]. The various modifications of the channel model and the existing numerical solutions [4-6] are based on the assumption of the predominant role played by molecular heat conduction in the mechanism of Joule heat transfer from the arc to the gas streamlining the arc, i.e., the flow of the gas in the discharge chamber is assumed to be laminar. This assumption is valid for long cylindrical arcs involving weak flow of gas, but it is hardly suitable in analyzing the characteristics of real heaters with gas-stabilized arcs, in which the longitudinal gradients for the parameters are substantial and in which the gas flow may be turbulent.

The turbulent (or jet) model of a longitudinally streamlined arc [7, 8] is based on the analogy between the propagation of a turbulent nonisothermal gas jet in a cocurrent flow and the development of an arc discharge stabilized by a gas flow in a cylindrical channel. Analysis of the longitudinal variations in the arc and flow parameters is facilitated through the introduction of a universal radial velocity profile, characteristic of turbulent jets [9]:

$$\frac{\Delta \bar{u}}{\Delta \bar{u}_m} = (1-\eta^{3/2})^2 = f(\eta). \tag{1}$$

Here $\eta = r/b$ is the dimensionless radial coordinate in the zone in which the jet mixes with the cocurrent flow. The discharge zone is separated from the unperturbed cold gas flow by a boundary whose radial coordinate b(x) is determined from the equation

$$\frac{db}{dx} = c\,\frac{u_m - u_0}{u_a}. \tag{2}$$

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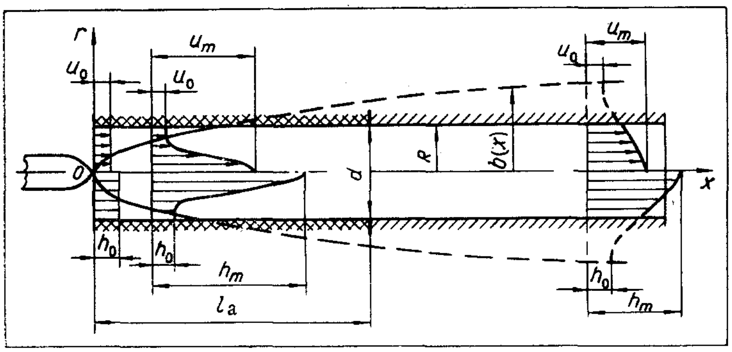

Fig. 1. Diagram of the turbulent (jet) model of a longitudinally streamlined arc.

The current–voltage characteristics for an arc of self-adjusting length in an atmosphere of argon, nitrogen, hydrogen, air, and helium were calculated in [7] on the basis of the turbulent arc model. The calculation was performed with the functions b(x) derived for an immersed isothermal jet and for the wake behind a poorly streamlined body. Equation (2) from [8] is used in general form, with the constant c assumed to equal 0.1 [9], and the characteristic velocity u_a determined by averaging and retention of the flow rate. No consideration was given in the cited references to the energy losses in the walls of the discharge channel as a consequence of radiation and convection. Below we present the solution for the problem relating to the determination of the electrical and gasdynamic gas-flow parameters in the stabilization channel of an electric-arc heater, with consideration given to the convection transfer of the heat between the gas and the channel wall that is being cooled. The magnitude of the specific heat flow to the channel wall is determined by the Kutateladze–Leont'ev [10] limit heat-transfer law for the turbulent boundary layer of a compressible gas on a plate (Pr = 1):

$$q_w = 0.029 \cdot \mathrm{Re}_x^{-0.2} \left(\frac{2}{1+\sqrt{\bar{h}_w}} \right)^{1.6} \rho_0 u_0 (h_1 - h_w). \tag{3}$$

The following assumptions have been made in the formulation of the problem:

1. The flow of the gas through the channel of the heater (Fig. 1) is turbulent, steady, one-dimensional, subsonic, and axisymmetric.

2. The plasma in the arc discharge is quasineutral and is in thermodynamic equilibrium.

3. There are no magnetic fields.

4. Heat losses from the arc-discharge region as a consequence of radiation are neglected [3].

5. The convection heat flow to the channel wall is determined from (3). The effect of the boundary layer on the structure of the flow in the channel is not taken into consideration.

6. The strength of the electric field is independent of the radial coordinate and is equal to E(x) = dV/dx.

7. We neglect the change in the pressure along the channel.

8. The velocity and enthalpy fields are similar and are determined from profile (1).

9. The channel wall does not affect the shape of profile (1) when b > R.

The equations are as follows:

$$\rho_0 u_0 \pi R^2 = \int_0^R 2\pi\rho u r dr = G, \tag{4}$$

$$(p_0 - p)\pi R^2 = \int_0^R 2\pi\rho u^2 r dr - \rho_0 u_0^2 \pi R^2, \tag{5}$$

$$i\frac{dV}{dx} = \frac{d}{dx}\left(\int_0^R 2\pi\rho u h r dr\right) + 2\pi R q_w, \tag{6}$$

$$i \int_0^R 2\pi\sigma \frac{dV}{dx} rdr, \tag{7}$$

$$\rho h = \text{const.} \tag{8}$$

The boundary conditions are the following:

$$x = 0:\ V = V_0;\ p = p_0;\ h = h_0\delta(r);\ u = u_0\,\delta(r). \tag{9}$$

Specifying the boundary conditions for the velocity and the enthalpy in the form of delta-functions is explained by the following circumstances: the jet of an intensively heated gas which is generated near a rod electrode, because of inadequate data to permit proper choice of its initial dimensions, is treated as a point source of finite intensity (Fig. 1). The infinite enthalpy at the pole O provides for an infinite current density.

At the section $x = l_a$ the arc closes on the wall of the cylindrical channel, and when $x > l_a$ $dV/dx = 0$. The arc length l_a in plasmatrons with an interelectrode insert can be assumed to be equal to the length of the insert. If the channel is formed by the second electrode, it becomes necessary to impose the breakdown condition. It was assumed in [8] that the breakdown occurs either at $b = R$, or at the section in which the electrical conductivity of the gas near the wall is equal to $1/\Omega \cdot m$.

Equation (4), in conjunction with the equation of state in the form of (8) and with the condition of similarity for the velocity and enthalpy fields, yields

$$\rho u = \rho_0 u_0 = \text{const.}$$

The momentum equation (5) may be utilized to evaluate the variations in pressure along the discharge channel:

$$p_0 - p = \rho_0 u_0^2 \Delta\bar{h}_a.$$

The integral of the energy equation (6), after introduction of the dimensional complexes, i.e.,

$$I = \frac{i^2}{Gh_0 d};\quad U = \frac{(V - V_0)\,d}{i}$$

assumes the form ($\bar{x} = x/R$; $h_w \approx h_0$)

$$IU = \Delta\bar{h}_a + \int_0^{\bar{x}} 0.058\,\mathrm{Re}_x^{-0.2} \left(\frac{2\sqrt{\bar{h}_1}}{\sqrt{\bar{h}_1} + 1}\right)^{1.6} \Delta\bar{h}_1 d\bar{x}. \tag{10}$$

The total-current equation (7) can be modified as follows:

$$\frac{dU}{d\bar{x}} = \frac{2\xi^2}{\pi\sigma_0 \Delta\bar{h}_m^{\alpha} \Psi(\xi)}. \tag{11}$$

The average electrical conductivity of the gas in this cross section of the channel is equal to

$$\sigma_a = \xi^{-2} \int_0^{\xi} 2\sigma\eta d\eta = \xi^{-2}\Psi(\xi) \int_0^1 2\sigma\eta d\eta,$$

and the quantity $\varphi = \int_0^1 2\sigma\eta d\eta$ for a specified radial enthalpy profile is a function of the single variable $\Delta\bar{h}_m$ and is approximated by the piecewise-smooth power function [8]

$$\varphi = \sigma_0 \Delta\bar{h}_m^{\alpha}. \tag{12}$$

To determine the dimensionless coordinate of the panel wall, i.e., $\xi(x) = R/b(x)$, we should use (2). Let us introduce the approximation of the right-hand member of this equation by a power function of the variable $\Delta\bar{h}_m$

$$\frac{d\xi}{d\bar{x}} = -\xi^2 B \Delta\bar{h}_m^{\beta}. \tag{13}$$

The quantities B and β are functions of the shape of the assumed enthalpy profile $f(\eta)$ for profile (1) we have B = 0, 1; B = 0.5. The dimensionless gas enthalpy $\Delta\bar{h}_m$ on the axis is associated with the mean-mass enthalpy by the relationship

$$\Delta\bar{h}_a = \xi^{-2}\int_0^{\xi} 2f(\eta)\,\eta d\eta\Delta\bar{h}_m = \Delta\bar{h}_m\Phi(\xi). \tag{14}$$

From (11) and (13) we obtain

$$\frac{dU}{d\xi} = -\frac{2}{\pi B\sigma_0\Delta\bar{h}_m^{\alpha+\beta}\Psi(\xi)}. \tag{15}$$

To integrate (15) we must express $\Delta\bar{h}_m$ in terms of U by means of (10) and (14). In (10) we make the transition from Re_x to Re_{wd}, as we assume that $\bar{x}^{0.2} = 1.6$ (the average value for $\bar{x}$ = 5-20 and let us introduce the approximation

$$\left(\frac{2\sqrt{\bar{h}_1}}{\sqrt{\bar{h}_1}+1}\right)^{1.6}\left(\frac{\mu_1}{\mu_w}\right)^{0.2} = 1.39\Delta\bar{h}_1^{0.13+0.2\omega}.$$

Since $\Delta\bar{h}_1 = \Delta\bar{h}_m f(\xi)$, we can write

$$IU = \Delta\bar{h}_a + \frac{0.058}{B\,Re_{wd}^{0.2}}\int_{\xi}^{\infty}\left(\frac{\Delta\bar{h}_a}{\Phi}\right)^{1.13+0.2\omega-\beta} f^{1.13+0.2\omega}\xi^{-2}d\xi.$$

Calculations of the mean-mass enthalpy without consideration of convection losses [8] showed that this quantity in the region $\xi < 1$ (b > R) varies rather weakly; therefore, the factor $\Delta\bar{h}_a^{1.13+0.2\omega-\beta}$ can be removed from the integral sign. The exponent $1.13 + 0.2\omega - \beta$ differs little from unity; bearing this in mind, we replace $\Delta\bar{h}_a^{1.13+0.2\omega-\beta}$ by $\Delta\bar{h}_a$. These rough assumptions enable us to derive a simple linear relationship between $\Delta\bar{h}_a$ and U.

Let us denote

$$\eta_k(\xi) = \left[1 + \frac{0.058}{B\,Re_{wd}^{0.2}}\int_{\xi}^{\infty} f^{1.13+0.2\omega}\Phi^{-1.13-0.2\omega+\beta}\,\xi^{-2}d\xi\right]^{-1}$$

and let us express $\Delta\bar{h}_a$ in terms of U

$$\Delta\bar{h}_a = IU\eta_k(\xi). \tag{16}$$

We can now impart the following form to Eq. (15):

$$\frac{dU}{d\xi} = -\frac{2\Phi^{\alpha+\beta}}{\pi B\sigma_0(IU\eta_k)^{\alpha+\beta}\Psi}. \tag{17}$$

Let us integrate (17) with the boundary condition U = 0 when $\xi = \infty$:

$$U = [F(\xi)]^{\frac{1}{\alpha+\beta+1}} I^{-\frac{\alpha+\beta}{\alpha+\beta+1}}. \tag{18}$$

Here we have denoted

$$F(\xi) = \frac{2(\alpha+\beta+1)}{\pi B\sigma_0}\int_{\xi}^{\infty}\left(\frac{\Phi}{\eta_k}\right)^{\alpha+\beta}\Psi^{-1}d\xi. \tag{19}$$

Using (14) and (16) we find the relationship between $\Delta\bar{h}_m$, $\Delta\bar{h}_a$, and ξ. It remains but to express ξ in terms of $\bar{x}$, i.e., to integrate (13):

$$\bar{x} = X(\xi) I^{-\frac{\beta}{\alpha+\beta+1}}, \tag{20}$$

where

$$X(\xi) = \frac{1}{B}\int_{\xi}^{\infty}\left(\frac{\Phi}{\eta_k}\right)^{\beta} F^{-\frac{\beta}{\alpha+\beta+1}}\,\xi^{-2}d\xi. \tag{21}$$

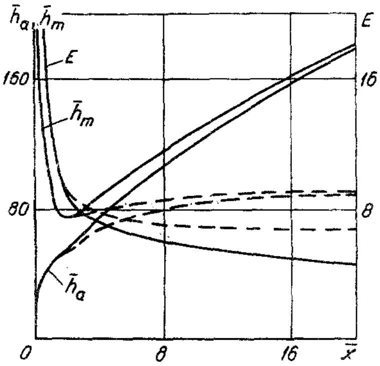

Fig. 2

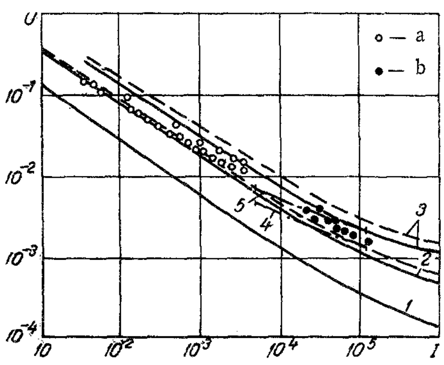

Fig. 3

Fig. 2. Distribution of the quantities $\bar{h}_a$, $\bar{h}_m$, and E over the length of the discharge channel of an argon arc heater (solid lines show $Re_{wd} = \infty$, with heat transfer not taken into consideration; the dashed lines are $Re_{wd} = 10^4$, with heat transfer taken into consideration).

Fig. 3. Comparison of theoretically and experimentally generalized U–I characteristics for an argon heater: 1) $\bar{l}_d = \bar{l}$ ($\xi = 1$) for an arc of self-adjusting length; 2) $\bar{l}_a = 10$; 3) $\bar{l}_a = 30$; 4) $\bar{l}_a = 10$; 5) $\bar{l}_a = 22$; 4 and 5 are based on Eq. (22); a) taken from [11]; b) taken from [12]; the solid lines are for $Re_{wd} = \infty$; the dashed lines are for $Re_{wd} = 10^4$.

In parametric form Eqs. (18) and (20) determine the function $U(\bar{x})$. Having specified the arc length l_a and the values of determining parameters I and Re_{wd}, we can calculate the generalized current–voltage characteristic of the arc.

The theoretical current–voltage characteristics for arcs of self-adjusting length are not subject to the effects of convection heat transfer to the wall, since according to the diagram (Fig. 1), the arc ends at the section b = R or at some section that is near by. Satisfactory agreement between theoretically and experimentally generalized current–voltage characteristics for an arc of self-adjusting length was achieved in [7, 8] for flows of various gases. Characteristics of heaters with an interelectrode insert depend strongly on Re_{wd}. This is borne out by the theoretical relationship between the strength of the longitudinal electrical field E, the mean-mass dimensionless enthalpy $\bar{h}_a$, and the dimensionless enthalpy $\bar{h}_m$ on the axis, as these relate to the length of the argon heater discharge channel, calculated for values of $I = 10^5$ (it is assumed for the calculation of E that d = 1 cm and G = 1 g/sec); this information is shown in Fig. 2. The calculation results demonstrate that with heat transfer ($Re_{wd} = 10^4$) the quantities $\bar{h}_a$ and $\bar{h}_m$ increase very rapidly along the length of the channel, approaching some equilibrium value. Hence it follows that the excessive elongation of the channel makes no sense at all, since a greater portion of the channel will operate at an efficiency close to 0. The losses of heat in the wall lead to an increase in the strength, and this is particularly noticeable if the interelectrode insert is very long.

Figure 3 – in generalized U–I coordinates – shows a comparison of the theoretical current–voltage characteristics for an arc in an argon atmosphere with the experimental data derived by various authors in dc heaters with interelectrode inserts. (In the comparison it is assumed that $V_0 = 0$.) The heater described in [11] had an interelectrode insert with a diameter of 25 mm and 200 mm in length ($\bar{l} = 16$); the data were derived for the following range of variations in the basic parameters: G = 1.5-5 g/sec ($Re_{wd} = 4 \cdot 10^3 - 1.3 \cdot 10^4$), i = 25-200 A. The studies were performed in [12, 13] on an electric-arc heater with a segmented discharge channel whose diameter was 18 mm, with the relative length $\bar{l}$ bearing from 6.0 to 22. The argon flow rate is 1.2-6 g/sec ($Re_{wd} = 4 \cdot 10^3 - 2 \cdot 10^4$), and the current strength is i = 100-600 A. The generalized heater characteristics [13] have the form

$$U = AI^{-0.38}\, Re_{wd}^{-0.27}\, \bar{l}^{\,0.65}. \tag{22}$$

Figure 3 shows the generalized characteristics of this heater for $Re_{wd} = 10^4$ and $\bar{l} = 10$; 20.

The calculation demonstrates qualitatively a series of phenomena that are familiar from experiments: a rise in voltage at the arc under the influence of the interelectrode insert (for purposes of comparison, we show the theoretical current–voltage characteristics for an arc of self-adjusting length); a change in the slope of the current–voltage characteristics and the transition from falling to rising V–i characteristics; the relationship between arc parameters and the Reynolds number and the relative length of the interelectrode insert.

NOTATION

u	is the velocity;
ρ	is the density;
p	is the pressure;
h	is the enthalpy;
μ	is the viscosity;
σ	is the electrical conductivity;
i	is the current strength;
V	is the potential of the electric field;
E	is the electric field strength;
G	is the gas flow rate through the heater;
σ_0 and α	are the coefficients in Eq. (12);
B and β	are the coefficients in Eq. (13);
ω	is the exponent in the formula $\mu \sim h^{\omega}$;
$\bar{u} = u/u_0$ and $\bar{h} - h/h_0$	are the dimensionless velocity and enthalpy;
$\Delta\bar{u} = \bar{u} - 1$, $\Delta\bar{h} = \bar{h} - 1$	are the dimensionless excess velocity and enthalpies;
$\bar{h}_w = h_w/h_1$	is the enthalpy factor;
$Re_x = \rho ux/\mu_1$, $Re_{wd} = \rho ud/\mu_w$	are the Reynolds numbers;
I and U	are the generalized dimensional criteria;
$\xi(x) = R/b$	is the relative coordinate of the channel wall.

Subscripts

0 denotes the parameters in the inlet of the channel;
m denotes the parameters at the channel axis;
w denotes the parameters at the wall temperature;
1 denotes the parameters at the wall (at the boundary-layer edge);
a denotes averaged parameters.

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